

APPM 1340

Exam 2

Fall 2024

Name		
Instructor	Richard McNamara	Section 150

This exam is worth 100 points and has **4 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and simplify your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

L'Hôpital's Rule and dominance of powers arguments can **NOT** be used to evaluate limits on this exam

1. (19 pts) Parts (a) and (b) are not related.

(a) For $f(x) = \frac{1}{x^2 - 4}$ and $g(x) = \frac{1}{x}$, identify the composite function $(f \circ g)(x)$ and its domain.

Express the domain in interval form.

(b) The graph of $y = \cos x$ is transformed in the following three steps, in the specified order:

- i) Stretched horizontally by a factor of 2
- ii) Shifted horizontally by 3 units to the right
- iii) Reflected across the x -axis

After each of the three transformations, what is the equation of the resulting graph? Note that no actual graphing is required in this problem.

i. Equation of the graph after transformation (i):

ii. Equation of the graph after transformations (i) and (ii):

iii. Equation of the graph after transformations (i), (ii), and (iii):

2. (31 pts) Evaluate the following limits. If you use a named theorem, state the name as part of your solution.

(a) $\lim_{x \neq 0} \frac{x \cot(3x)}{x - 4}$

(b) $\lim_{x \neq 4} \frac{x + 4}{x^2 + 9} \cdot 5$

$$(c) \lim_{x \rightarrow 2} (x - 2)^2 \sin \frac{1}{x - 2}$$

3. (32 pts) Consider the function $h(x) = \frac{(x-1)\sqrt{4x^4+1}}{x^3(3x^2+2x)}$.

(a) Identify all values of x , if any, for which $y = h(x)$ has a removable discontinuity. Support your answer by evaluating the appropriate limit(s).

(b) Find the equation of each vertical asymptote of $y = h(x)$, if any exist. Support your answer by evaluating the appropriate limit(s).

- (c) Find the equation of each horizontal asymptote of $y = h(x)$, if any exist. Support your answer by evaluating the appropriate limit(s). (*Reminder: You may not use L'Hopital's Rule or dominance of powers arguments to evaluate limits on this exam.*)

4. (18 pts) Parts (a) and (b) are not related.

(a) For what pair of values a and b is the following function $u(x)$ continuous at $x = 3$? Support your answer using the definition of continuity, which includes evaluating the appropriate limits.

$$u(x) = \begin{cases} x + \frac{a}{x} & ; \quad x < 3 \\ 2b + 2 & ; \quad x = 3 \\ x + b & ; \quad x > 3 \end{cases}$$

- (b) Use a Calculus 1 theorem to establish that the equation $v(x) = (x - 1)(x + 2) + \sin^2 x = 0$ has at least one solution on the interval $(0; -2)$. Name the theorem that is used and verify that the conditions for applying it to this problem are satisfied.

END OF EXAM

Your Initials _____

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.