

1. (36 pts) Evaluate the integral.

$$(a) \int \left(\tan x + \frac{1}{\cos x} \right)^2 dx \quad (b) \int \frac{11}{(2x-1)(3x+4)} dx \quad (c) \int \frac{3x^3 + 18x - 1}{x^2 + 6} dx$$

Solution:

(a)

$$\begin{aligned} \int \left(\tan x + \frac{1}{\cos x} \right)^2 dx &= \int (\tan x + \sec x)^2 dx \\ &= \int \left(\tan^2 x + 2 \tan x \sec x + \sec^2 x \right) dx \\ &= \int \left(\frac{\sec^2 x - 1}{\sec^2 x} + 2 \tan x \sec x + \sec^2 x \right) dx \\ &= \int \left(2 \sec^2 x - 1 + 2 \tan x \sec x \right) dx \\ &= \boxed{2 \tan x + 2 \sec x + C} \end{aligned}$$

(b)

$$\int \frac{11}{(2x-1)(3x+4)} dx = \int \left(\frac{A}{2x-1} + \frac{B}{3x+4} \right) dx$$

The coefficients are $A = 2$ and $B = 3$

$$\begin{aligned} &= \int \left(\frac{2}{2x-1} + \frac{3}{3x+4} \right) dx \\ &= \boxed{\ln |2x-1| + \ln |3x+4| + C} \end{aligned}$$

(c)

$$\begin{aligned} \int \frac{3x^3 + 18x - 1}{x^2 + 6} dx &= \int \left(3x + \frac{1}{x^2 + 6} \right) dx \\ &= \frac{3}{2} x^2 + \frac{1}{\sqrt{6}} \arctan \left(\frac{x}{\sqrt{6}} \right) + C \end{aligned}$$

$$Z = 2$$

2. (26 pts) Consider the integral $\int_0^2 \cos(x) dx$.

- (a) Estimate the integral using the trapezoidal approximation. Fully simplify your answer.
- (b) Find error estimate E_j for the approximation. You may leave your answer unsimplified. Hint: The first derivative of $\cos(x)$ is $-\sin(x)$.
- (c) Find the f

3. (16 pts) The shaded region shown below is bounded by $y = \sin(x)$ and $y = x \sin(x)$. The region is composed of two smaller regions R_1 above the x -axis and R_2 below the x -axis. Set up (but do not evaluate) integrals to find the following quantities.
- The area of shaded region which lies above the x -axis
 - The volume of the solid generated by rotating the entire shaded region (both R_1 and R_2) about the line $x = 2$

Solution:

The curve $y = x \sin(x)$

