

Friday,

This exam has 4 problems. Show all your work. Justification will receive no points. You are allowed a calculator, smartphone, smartwatch, the Internet

**Problem 1** (30 pts)

Consider the function

- Graph the level curve of  $f(x; y)$  that passes through the point  $(0; 2)$ .
- On the same graph as part (a) graph the level curve  $f(x; y) = 1$ .
- On the same graph as part (a), graph one level curve where  $f(x; y) < 0$ .
- At the point  $(1; 1)$ , give a vector that points in the direction of steepest increase.
- Sketch the vector you found in part (d) starting at the point  $(1; 1)$ .
- Use a 2nd order (i.e. quadratic) Taylor approximation to approximate  $\frac{\sqrt{1.8}}{1.5}$ . You can leave your answer as an unsimplified expression.

**Problem 2** (22 pts) The temperature (in degrees Celsius) in a region in space is given by

$$T(x, y, z) = 1 - \frac{1}{2}xyz$$

A particle is moving in this region and its position vector is given by

$$\mathbf{r}(t) = \langle e^{9-t}, \dots \rangle$$

**-Exam 2**

1-2:35pm 2022

your answers. Answers with missing or insufficient justification will receive no points. You are allowed a calculator, smartphone, smartwatch, the Internet, and a graphing calculator. You may NOT use a computer or any other electronic device.

Consider the function  $f(x; y) = 1 - x^2 - y^2$ . The domain of  $f$  is the region in space where  $f(x; y) > 0$ .

(a) Graph the level curve of  $f(x; y)$  that passes through the point  $(0; 2)$ . Label the value of  $f$  along the curve.

(b) On the same graph as part (a) graph the level curve  $f(x; y) = 1$ . Label the value of  $f$  along this curve.

(c) On the same graph as part (a), graph one level curve where  $f(x; y) < 0$ . Label the value of  $f$  along this curve.

(d) At the point  $(1; 1)$ , give a vector that points in the direction of steepest increase. Label the value of  $f$  along this curve.

(e) Sketch the vector you found in part (d) starting at the point  $(1; 1)$ .

$$T(x, y, z) = 1 - \frac{1}{2}xyz$$

(f) A particle is moving in this region and its position vector is given by

$$\mathbf{r}(t) = \langle e^{9-t}, \dots \rangle$$

**Problem 4** (20 pts)

A mother puts her child on an amusement park ride that takes the child along a path in the  $xy$ -plane described by the equation  $x^2 - 2x = 4y - y^2$ . While the child is on the ride, the mother stands at the location  $(x; y) = (0; 0)$ .

- (a) Use Lagrange multipliers to find the minimum and maximum distances from the mother to the child during the ride.
- (b) Give the  $(x; y)$  coordinates of the child at the minimum and maximum distances.

---

End Of Exam

---