4. [2360/062422 (24 pts)] Let $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & 5 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Evaluate each of the following expressions or explain why it is not defined.

  (a) $\mathbf{AB}$ (b) $\mathbf{B} + 2\mathbf{I}$ (c) $\mathbf{A}^T\mathbf{A}^T$ (d) $jAjA^{-1}$ (e) $\mathbf{B}^T\mathbf{A}$ (f) $\text{Tr } \mathbf{B}^2$

5. [2360/062422 (20 pts)] The following parts are not related. However, you need to provide justification for all of your answers for each part. Correct answers with missing or incorrect justifications will receive no points.

  (a) (14 pts) Consider the vector space $\mathbb{R}^4$.

   i. (8 pts) Suppose $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are vectors in $\mathbb{R}^4$ and that the only solution to $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$ is $c_1 = c_2 = c_3 = 0$.
   A. (4 pts) Is span $f\vec{v}_1, \vec{v}_2, \vec{v}_3 g = \mathbb{R}^4$?
   B. (4 pts) Is $\vec{v}_1 \in$ span $f\vec{v}_2, \vec{v}_3 g$?

   ii. (6 pts) Let W be the set of vectors $\mathbb{R}^4$ of the form $\begin{bmatrix} a \\ b \\ 0 \\ ab \end{bmatrix}$