Write your name below. You must show your work and not give decimal answers (i.e. don't use a calculator or software to compute a decimal answer). You are not allowed to collaborate on the exam or seek outside help, though using your notes, the book, the recorded lectures, or material

(h) Suppose A is a symmetric matrix. Then, $Ax = b$ has a solution if and only if b is orthogonal to the corange of A.

(i) If A is invertible, then A is diagonalizable.

(j) The singular values of a nonsingular matrix A are the same as the singular values of A^{-1}.

2. Consider the following linear transformation: $L(x; y) = \begin{pmatrix} x + y \\ 2x \quad y \end{pmatrix}$.

   (a) (4 points) Find the matrix form of $L(x; y)$ with respect to the standard basis.

   (b) (6 points) Find the matrix form of $L(x; y)$ with respect to the following basis: $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. (The same basis is used for both the domain space and the co-domain space.)

3. (20 points) Find the Jordan decomposition of $A =$ 2 4 3 1 3 1 3 1

4. (20 points) Find the singular value decomposition of

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \end{pmatrix} :$$

7. (10 points) Find the least-squares solution of the system

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}.$$