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Interactions of dispersive shock waves (DSWs) and rarefaction waves (RWs) associated with the Korteweg–de Vries equation are shown to exhibit multiphase dynamics and isolated solitons. There are six canonical cases: one is the interaction of two DSWs that exhibit a transient two-phase solution but evolve to a single-phase DSW for large time; two tend to a DSW with either a small amplitude wave train or a finite number of solitons, which can be determined analytically; two tend to a RW with either a small wave train or a finite number of solitons; finally, one tends to a pure RW.

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Shock waves in processes dominated by weak dispersion and nonlinearity have been experimentally observed in plasmas [1], water waves [2], and more recently in Bose-Einstein condensates [3,4] and nonlinear optics [5]; these dispersive shock waves (DSWs) have yielded novel dynamics and in-

Although the (initial) shock front speed is different for DSWs and VSWs ($2\ _0/3$ and $\ _0$

In case IV , a small DSW forms on the left and a large RW forms on the right see Fig. 6a. As in case II , the front of the DSW interacts with the trailing edge of the RW and decreases the DSW's amplitude and speed. Unlike case II , the front of the DSW does not

6b

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