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## Search, Heterogeneity, and Optimal Income Taxation

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**Abstract**

[illegible]

[illegible]

D [illegible] . N [illegible] M [illegible] (1982), H [illegible] (1990) [illegible] A [illegible] (1999) [illegible] . H [illegible] (2001) [illegible] . I [illegible]

[illegible] . I [illegible] . I [illegible]

## 2 Model

$(H)$ $(L)$ $l_k$, $k = H; L$, $q_m$, $m = H; L$.

$y_{km} > 0$,

$y_{Hm} > y_{Lm}$.

$\delta_k \in [0; 1]$.

$c_w(\delta_k)$,

$c_w(0) = 0$, $c'_w(0) = 0$, $\lim_{\delta_k \to 1} c'_w(\delta_k) = +\infty$.

$v_m$, $c_\pi(v_m)$

## 2.1 The matching technology

, 1 – ff

. A M P (1999) ,

. H , P (1996) Bl D -

## 2.2 Output sharing

## 2.4 Private expected utility functions

fi , , $v_H q_H = \ _m q_m$.

$k$

$$U_k = -c_w(\ _k) + \ _k \ M(\ ) \ \frac{v_H q_H}{_m v_m q_m} \ _{kH} y_{kH} + \frac{v_L q_L}{_m v_m q_m} \ _{kL} y_{kL} \ + (1 - M(\ ))0 \ + (1 - \ _k)0$$

$$U_k = -c_w(\ _k) + \ _k M(\ ) E_{(m)} \ _{km} y_{km}; \tag{5}$$

$E_{(m)}$ . A , , $c(\ )$ , . O , , $M(\ )$, . I , $1 - M(\ )$, , , . I , (

## 3 Optimal search intensity and market inefficiencies

### 3.1 Social Optimum

$$W = \quad_{\delta,v} \quad \sum_k l_k U^k + \sum_m q_m V^m$$

$$. \; . \quad _k \geq 0; \quad v_m \geq 0:$$

(1), (2), (5), (6), -91.21

$$E_{(m)}y_{km} - (1 - \;)E_{(k)}E_{(m)}y_{km} = \; E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}.$$

A [illegible] $E_{(k)}E_{(m)}y_{km}$ [illegible] $E_{(m)}y_{km}$ [illegible] $E_{(k)}E_{(m)}y_{km}$. [illegible]

[illegible] 1. [illegible]

$$c'_{\pi}(\bar{v}_H) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kH} - \; E_{(k)}E_{(m)}y_{km}, \quad c'_{\pi}(\bar{v}_L) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kL} - \; E_{(k)}E_{(m)}y_{km} \quad \bar{\ } \lesseqgtr 1; \; \bar{v}_H > 0; \; \bar{v}_L > 0 \tag{13}$$

$$c'_{\pi}(\bar{v}_H) = \frac{M(\ )}{} E_{(k)}y_{kH} - \; E_{(k)}E_{(m)}y_{km}, \quad c'_{\pi}(0) \geq \frac{M(\ )}{} E_{(k)}y_{kL} - \; E_{(k)}E_{(m)}y_{km} \quad \bar{\ } \lesseqgtr 1; \; \bar{v}_H > 0; \; \bar{v}_L = 0 \tag{14}$$

## 3.2 Decentralized equilibrium

A [illegible] $k$ [illegible]

$$\max_{\delta_k} U_k = -c_w(\ _k) + \ _k M(\ )E_{(m)} \ _{km}y_{km} \quad \text{s.t.} \ _k \geq 0; \tag{15}$$

[illegible] $C_k = c_w(\ _k)$ [illegible], [illegible] $B_k = \ _k M(\ )E_{(m)} \ _{km}y_{km}$ [illegible] fi [illegible].

K [illegible] $k$ [illegible] 2

$$
\begin{aligned}
-c_w'(\ _k) + M(\ )E_{(m)}\ _{km}y_{km} &\leq 0 \\
_k &\geq 0 \qquad\qquad (16)\\
(-c_w'(\ _k) + M(\ )E_{(m)}\ _{km}y_{km})\ _k &= 0;
\end{aligned}
$$

[illegible] $_k$ [illegible], [illegible]. [illegible] $M(\ )$, [illegible]. F [illegible], [illegible] ff [illegible], [illegible] ff [illegible].

I [illegible] - [illegible]. [illegible] $_{LL} \neq 0$ ( [illegible] $_{LH} = 0$), [illegible] $y_{km} > 0$, [illegible] fi [illegible] (06 )2 158.730 ( ) -395. 4.9812163(0),)-373( )-333( ( )-350( 5. 4

fi . N , I P ( ) P . L , I : . I , I .

I $k$ ( $m$ ) ( ) , fi ( ) ( ) . B , ( ) . I , . ff - . I - , , . H , - , I . fi

$_H M(\ ) E_{(m)} w_{Hm}$. ( ) $\left( _L \frac{E_{(m)} w_{Lm}}{E_{(m)} w_{Hm}} \right.$ $M(\ ) E_{(m)} w_{Hm} =$ $_L M(\ ) E_{(m)} w_{Lm}$, $\left( _L \frac{E_{(m)} w_{Lm}}{E_{(m)} w_{Hm}} \right.$ $_L M(\ ) E_{(m)} w_{Lm}$ [3]. , $_H M(\ ) E_{(m)} w_{Hm}$ $_L M(\ ) E_{(m)} w_{Lm}$, ff : $^w_H$, $_H M(\ ) E_{(m)} w_{Hm}$, $^w_L$, $_L M(\ ) E_{(m)} w_{Lm}$.

[13]

. D l ll K (1992), ll B B (2002),

. O ... fi

**4.** *I*

$$c'_w(\ _k) = M(\ )(1 - \ ^w_k)w_k \qquad \lessgtr 1;$$

$$c'_\pi(v_m) = \frac{M(\ )}{}(1 - \ ^\pi_m)\ _m \qquad _k > 0;\ v_m > 0 \quad ; \tag{22}$$

$$c'_w(0) \geq M(\ )(1 - \ ^w_L)w_L \qquad \lessgtr 1;$$

$$c'_\pi(0) \geq \frac{M(\ )}{}(1 - \ ^\pi_L)\ _L \qquad _L = 0;\ v_L = 0 \quad : \tag{23}$$

## 4.1 Characterizing externalities through Pigou taxes

, P ... $(\check{\ }^w_H; \check{\ }^w_L)$, P ... $(\check{\ }^\pi_H; \check{\ }^\pi_L)$. L ... ( ... ) $\check{R}$, P , ... , $LS$:

$$\check{R} = (\ _k\ l)M(\ )\ \frac{_H l_H}{_k\ l}\check{\ }^w_H w_H + \frac{_L l_L}{_k\ l}\check{\ }^w_L w_L + \frac{v_H q_H}{_m vq}\check{\ }^\pi_H\ _H + \frac{v_L q_L}{_m vq}\check{\ }^\pi_L\ _L$$

$$0 = \check{R} - \sum_k l_k + \sum_m l_m\ LS;$$

$(\ _k\ l)M(\ ) = N$ ... .

... $k$

$$U_k = -c_w \frac{z^w_k}{M(\ )w_k} + LS + (1 - \check{\ }^w_k)z^w_k \tag{24}$$

, $LS$, ... ff

. fi ... ,

$c'_w$

[illegible] ( [illegible] P [illegible] 5)
[illegible] . CR [illegible], [illegible]
[illegible]; [illegible]
[illegible]
[illegible], [illegible] .
[illegible], [illegible] P [illegible]
[illegible], [illegible]
[illegible] . [illegible]
[illegible], [illegible]
[illegible] ff [illegible] . I[illegible]
[illegible] fi[illegible], [illegible] fi[illegible], L[illegible],
[illegible] .

N[illegible], [illegible]
[illegible] ff [illegible] $(g = 0)$. [illegible]
[illegible], [illegible]
[illegible]. I[illegible]

$y_{LL} > 0$, [illegible]

[illegible]

## 4.2 Optimal income taxes with positive government revenue

[illegible] $R$, [illegible]

$$W = \sum_k l_k U^k + \sum_m q_m V^m ;$$

[illegible]

[illegible] (1), (2), (5), [illegible] (6), [illegible] $w_k = E_{(m)} \; {}_{km} y_{km}$, $z_k^w = {}_k M( \; ) w_k$, ${}_m = E_{(k)}(1 - {}_{km}) y_{km}$, [illegible] $z_m^\pi = v_m \frac{M \theta}{\theta} \; {}_m$ [illegible] 4, [illegible]

$$W = \sum_k l_k \left( -c_w \frac{z_k^w}{M( \; ) w_k} \right) + \sum_m q_m \left( -c_\pi \frac{z_m^\pi}{\frac{M\theta}{\theta} \; {}_m} \right) + (\sum_k l) M( \; ) E_{(k)} E_{(m)} y_{km};$$

[illegible] $(\sum_k l) M( \; ) = N$ [illegible].

[illegible] R, [illegible]

$$R \leq (\sum_k l) M( \; ) \left( \frac{{}_H l_H}{\sum_k l} {}_H^w w_H + \frac{{}_L l_L}{\sum_k l} {}_L^w w_L + \frac{v_H q_H}{\sum_m vq} {}_H^\pi \; {}_H + \frac{v_L q_L}{\sum_m vq} {}_L^\pi \; {}_L \right); \qquad (30)$$

[illegible] $(\sum_k l) M( \; ) = M$ [illegible] (30), [illegible]

17 [illegible]

ldld dldslsdzdHdzdHdzddkqwmldMdHddkdddHdddLddLdddkqwmldMdLddkisiddLqwdddHdzdHdzddkmqwdldMdHdddldkdlddddlHddqdddLddLdddkmqwdldMdLdd

$$W = \sum_k l_k \left( -c_w \frac{z_k^w}{M(\ )\, w_k} \right) + \sum_m q_m \left( -c_\pi \frac{z_m^\pi}{\frac{M\ \theta}{\theta}\ _m} \right)$$
$$+ (\ _k\ l) M(\ ) \left( \frac{_H l_H}{_k\ l}(1 - \ ^w_H) w_H + \frac{_L l_L}{_k\ l}(1 - \ ^w_L) w_L \right.$$
$$\left. + \frac{v_H q_H}{_m\ vq}(1 - \ ^\pi_H)\ _H + \frac{v_L q_L}{_m\ vq}(1 - \ ^\pi_L)\ _L \right) + R:$$

..., (22) ( C (1978), D (1998) ...).

H ...

L ... 7 ... A ... E ... (22) ...

$$\max_{\tau_k^w, \tau_m^\pi} W = \sum_k l_k \left( -c_w \frac{z_k^w}{M(\ )\, w_k} \right) + \sum_m q_m \left( -c_\pi \frac{z_m^\pi}{M\ \theta} \right.$$

P 8 A .

$''^{w}_{k}=0$ $''^{\pi}_{m}=0$. I (33). H ff

I P 5 A A). H

I fi, $''^{\pi}_{m}=\infty$,

I $''^{\pi}_{m}=\infty$,

ff

#### 4.2.2

I I 1

**12.**

$$i) \; @ \; \frac{w_H}{w_L} = @ \; \frac{E_{(m)} \quad _{Hm}}{E_{(m)} \quad _{Lm}} < 0 \tag{36}$$

$$ii) \; @ \; \frac{w}{\pi} = @( \;\; ) < 0 \tag{37}$$

[illegible] P [illegible] 11 [illegible] A [illegible] B [illegible] ( [illegible] P [illegible] 5). [illegible]

P [illegible] 12 [illegible] B [illegible] B [illegible] (2002) [illegible]

## 5 Conclusion

[illegible]

[illegible]


[illegible] (1[illegible]) "H[illegible] Effi[illegible] F[illegible]," *I[illegible] E[illegible]*, 40, 827-849.

[illegible] (2002) "O[illegible] L[illegible]," *J[illegible] E[illegible]*, 85, 53-97.

[illegible] (1[illegible]4) "Eff[illegible] B[illegible]fi[illegible] F[illegible]," [illegible] P[illegible].

[illegible] (1[illegible]) "M[illegible] C[illegible]," *J[illegible] E[illegible]*, 112, 141-168.

[illegible] (1[illegible]) "L[illegible]-[illegible] P[illegible] F[illegible]: M[illegible] E[illegible]," *E[illegible] J[illegible]*, 109, 307-334.

[illegible] (1[illegible]) "O[illegible] R[illegible]: M[illegible] R[illegible]," *A[illegible] E[illegible]*, 68, 756-768.

[illegible] (1[illegible]1) "M[illegible] C[illegible], F[illegible] Effi[illegible]," *J[illegible] E[illegible]*, .89, 798-812.

[illegible] (1[illegible]2) "A[illegible] D[illegible] M[illegible] E[illegible]," *J[illegible] E[illegible]*, .90, 881-894.

[illegible] (1[illegible]2) "[illegible] D[illegible] Effi[illegible] E[illegible]," *E[illegible]*, .49, 217-227.

[illegible] (1[illegible]) "O[illegible] I[illegible]: A[illegible] E[illegible] P[illegible] O[illegible] M[illegible] R[illegible]," *A[illegible] E[illegible]*, .88, 83-95.

[illegible] (1[illegible]) "[illegible] P[illegible]," *E[illegible] J[illegible]*, 7, 46-70.

[illegible] (1[illegible]3) "O[illegible] O[illegible] P[illegible] I[illegible]," *J[illegible] E[illegible]*, 2, 357-376.

[illegible] (1[illegible]) "H[illegible] D[illegible] P[illegible] Aff[illegible] E[illegible]? C[illegible] O[illegible] E[illegible]," [illegible] M[illegible] P[illegible].

, . (1 0) "Effi I ," *J E* , 51, 8999.

(1 5) "P , , , P , D E , .

, . . (1 0) "O Effi M R M ," *E* , 57, 279-298.

, . (1 ) " , F , ," *E* , 32, D E , .

, ., . (1 ) " P G E P M B ," *L E* , 3, 6580.

., . (1 ) "L R , G J B J ," *E* , *C* , P .

, ., . (1 3) "

(1 ) "L M D Eff , J E , 33, 457495.

(2000) "A M ," E , 68, 343-369.

(2001) "M , , H " A , 1(2001), I . 1, A . 5.

. (1 ) "P F E P ?," E , 25, 223264.

. (1 ) "O P F L M ," L E , 6, 435452.

. (1 ) "P Effi O N N E ," I : H E , .2.

. (1 ) " - P Effi ," J E , 6, 239262.

. (1 ) " , P P D .," I F , 6, 239262.

(1 ) " P F E ," I : , .B. (E .), F C ., M M , L .

# Appendices:

## A Proofs of the main results

*Proof of Corollary 3.*

F , . -

, $_1 \neq$ . I $_1 >$ ,

$_H(_1) > _H()$, $_L(_1) > _L()$, $v_H(_1) < v_H()$, $v_L(_1) < v_L()$. H

$)_1$ )

, CR

. F $1-$ $=$

$$\check{R} = N \left[ \begin{array}{l} \frac{\delta_H l_H}{k\,\delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\,\delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\,vq} (1 - (1 - \quad)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad_{kH} y_{kH} \\ + \frac{v_L q_L}{m\,vq} (1 - (1 - \quad)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad_{kL} y_{kL} \end{array} \right]$$

$$= N \left[ \begin{array}{l} \frac{\delta_H l_H}{k\,\delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\,\delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\,vq} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad_{kH} y_{kH} \\ + \frac{v_L q_L}{m\,vq} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad_{kL} y_{kL} \end{array} \right]$$

$$\check{R} = N \quad 1 - ( \quad + \quad )$$

; - , . I ( L ).□

*Proof of Lemma 7.*

I , $k$

$$U_k = \; - c_w ( \;\; _k) + \;\; _k M( \;\; )(1 - \;\; ^w_k) w_k$$
$$= \; - c_w \quad \frac{z^w_k}{M( \;\; ) \, w_k} \quad + (1 - \;\; ^w_k) z^w_k$$

$w_k$

$$\frac{@U_k}{@w_k} = -c'_w \frac{1}{M( \;\; ) \, w_k} \frac{@z^w_k}{@w_k} - c'_w \frac{z^w_k}{M( \;\; )} \quad -\frac{1}{w^2_k} \quad + \frac{@z^w_k}{@w_k} (1 - \;\; ^w_k) = c'_w \frac{z^w_k}{M( \;\; ) \, w_k} > 0;$$

E . □

*Proof of Proposition 8.*

I : $w_k = E_{(m)} w_{km} = E_{(m)} \quad {}_{km} y_{km}$

$k = H; L;\ z^w_k = \quad {}_k M(\quad) w_k$

$k = H; L;\quad {}_m = E_{(k)} \quad {}_{km} = E_{(k)} (1 - \quad {}_{km}) y_{km}$ fi

e $m = H; L;\quad z^\pi_m = v_m \, {}^M$ z

Z

π π

k

$$\frac{dz^{w}_{H}}{z^{w}_{H}} \quad \frac{1}{''^{w}_{H}} + (1 - \quad ) \frac{{}_{H} l_{H}}{k \quad l} \quad = (1 - \quad ) \quad E_{(m)} \quad \frac{dz^{\pi}_{m}}{z^{\pi}_{m}} \quad - \frac{{}_{L} l_{L}}{k \quad l} \quad \frac{dz^{w}_{L}}{z^{w}_{L}} \quad - \frac{d \quad {}^{w}_{H}}{1 - \quad {}^{\pi}} \quad ($$

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{''^\pi_H} = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad + \frac{d\tau_L^\pi}{-\tau_L^\pi} \quad \frac{v_L q_L}{_m\, vq}''^\pi_L - \frac{d\tau_H^\pi}{-\tau_H^\pi}\Big(1 + \quad \frac{v_L q_L}{_m\, vq}''^\pi_L}{\Delta_2} \tag{45}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{''^\pi_L} = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad - \frac{d\tau_L^\pi}{-\tau_L^\pi}\Big(1 + \quad \frac{v_H q_H}{_m\, vq}''^\pi_H \quad + \frac{d\tau_H^\pi}{-\tau_H^\pi} \quad \frac{v_H q_H}{_m\, vq}''^\pi_H}{\Delta_2}; \tag{46}$$

$\Delta_2 = 1 + \quad E_{(m)}''^\pi_m$. A (45) (46)

$$E_{(m)} \quad \frac{dz_m^\pi}{z_m^\pi} \quad = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad E_{(m)}''^\pi_m - E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2}; \tag{47}$$

(43) (44)

$$E_{(k)} \quad \frac{dz_k^w}{z_k^w} \quad = \frac{(1 - \quad )E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi} \quad E_{(k)}''^w_k - E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w}}{\Delta_1}: \tag{48}$$

F (47) (48) $E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi}$ $E_{(k)}\Big(\frac{dz_k^w}{z_k^w}$

, ,

$$E_{(m)} \quad \frac{dz_m^\pi}{z_m^\pi} \quad = - \frac{(\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1 + \Delta_2 - 1} \tag{49}$$

$$E_{(k)} \quad \frac{dz_k^w}{z_k^w} \quad = - \frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1 + \Delta_2 - 1} \tag{50}$$

(49) (43) (44), (50) (45) (46) fi /

:

$$\frac{dz_H^w}{z_H^w}\frac{1}{''^w_H} = -\frac{(1 - \quad ) \quad (\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$
$$+ \frac{(\Delta_1 + \Delta_2 - 1) \quad \frac{d\tau_L^w}{-\tau_L^w}(1 - \quad )\frac{\delta_L l_L}{_k\, \delta l}''^w_L - \frac{d\tau_H^w}{-\tau_H^w}\Big(1 + (1 - \quad )\frac{\delta_L l_L}{_k\, \delta l}''^w_L}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{51}$$

$$\frac{dz_L^w}{z_L^w}\frac{1}{''^w_L} = -\frac{(1 - \quad ) \quad (\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$
$$+ \frac{(\Delta_1 + \Delta_2 - 1) \quad - \frac{d\tau_L^w}{-\tau_L^w}\Big(1 + (1 - \quad )\frac{\delta_H l_H}{_k\, \delta l}''^w_H \quad + \frac{d\tau_H^w}{-\tau_H^w}(1 - \quad )\frac{\delta_H l_H}{_k\, \delta l}''^w_H}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{52}$$

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{\text{"}_H^\pi} = -\frac{\Delta_2 E_{(k)}\Big(\text{"}_k^w \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big(\text{"}_m^\pi \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)} + \frac{(\Delta_1+\Delta_2-1) \quad \frac{d\tau_L^\pi}{-\tau_L^\pi} \quad \frac{v_L q_L}{_k\, vq}\text{"}_L^\pi - \frac{d\tau_H^\pi}{-\tau_H^\pi}\Big(1 + \quad \frac{v_L q_L}{_k\, vq}\text{"}_L^\pi}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{53}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{\text{"}_L^\pi} = -\frac{\Delta_2 E_{(k)}\Big(\text{"}_k^w \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big(\text{"}_m^\pi \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)} + \frac{(\Delta_1+\Delta_2-1) \quad -\frac{d\tau_L^\pi}{-\tau_L^\pi}\Big(1 + \quad \frac{v_H q_H}{_k\, vq}\text{"}_H^\pi \quad + \frac{d\tau_H^\pi}{-\tau_H^\pi} \quad \frac{v_H q_H}{_k\, vq}\text{"}_H^\pi}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{54}$$

F [illegible] (51)-(54) [illegible]

$$\begin{aligned}
\frac{dz_H^w}{d\ {}_H^w}\frac{1}{z_H^w} &= \frac{\text{"}_H^w}{1-\ {}_H^w} \quad \frac{(1- \quad)\frac{\delta_H l_H}{_k\,\delta l}\text{"}_H^w - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_H^w}\frac{1}{z_L^w} &= \frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}\text{"}_L^w(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}_H^w}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}\text{"}_H^\pi}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}_H^w}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}\text{"}_L^\pi}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{55}$$

$$\begin{aligned}
\frac{dz_H^w}{d\ {}_L^w}\frac{1}{z_H^w} &= \frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}\text{"}_H^w(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_L^w}\frac{1}{z_L^w} &= \frac{\text{"}_L^w}{1-\ {}_L^w} \quad \frac{(1- \quad)\frac{\delta_L l_L}{_k\,\delta l}\text{"}_L^w - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}_L^w}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}\text{"}_H^\pi}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}_L^w}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}\text{"}_L^\pi}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{56}$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}_H^\pi}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\ {}''^w_H(1- \ )}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_H^\pi}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\ {}''^w_L(1- \ )}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}_H^\pi}\frac{1}{z_H^\pi} &= \frac{{}''^\pi_H}{1- \ {}_H^\pi}\ \frac{{}''^\pi_H\frac{v_H q_H}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}_H^\pi}\frac{1}{z_L^\pi} &= \frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\ {}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}_L^\pi}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\ {}''^w_H(1- \ )}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_L^\pi}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\ {}''^w_L(1- \ )}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}_L^\pi}\frac{1}{z_H^\pi} &= \frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\ {}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}_L^\pi}\frac{1}{z_L^\pi} &= \frac{{}''^\pi_L}{1- \ {}_L^\pi}\ \frac{{}''^\pi_L\frac{v_L q_L}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{58}
$$

[illegible]

I [illegible], [illegible] . [illegible] L [illegible], [illegible] , [illegible]

$$
\begin{aligned}
\max_{\tau_k^w,\tau_m^\pi} W = & \sum_k l_k \ -c_w\ \frac{z_k^w}{M(\ )\,w_k} \ + \sum_m q_m \ -c_\pi\ \frac{z_m^\pi}{\frac{M\ \theta}{\theta}\ {}_m}\\
& + {}_H l_H c_w'\ \frac{z_H^w}{M(\ )\,w_H} \ + {}_L l_L c_w'\ \frac{z_L^w}{M(\ )\,w_L} \ + v_H q_H c_\pi'\ \frac{z_H^\pi}{\frac{M\ \theta}{\theta}\ {}_H} \ + v_L q_L c_\pi'\ \frac{z_L^\pi}{\frac{M\ \theta}{\theta}\ {}_L} \ + R\\
& + (\ {}_k\ l)M(\ )\ \frac{{}_H l_H}{{}_k\ l}\ {}_H^w w_H + \frac{{}_L l_L}{{}_k\ l}\ {}_L^w w_L + \frac{v_H q_H}{{}_m vq}\ {}_H^\pi\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}_L^\pi\ {}_L\ ;
\end{aligned}
$$

[illegible] . D[illegible]

$$
a = \frac{{}_H l_H}{{}_k\ l}\ {}_H^w w_H + \frac{{}_L l_L}{{}_k\ l}\ {}_L^w w_L \quad [\text{illegible}] \quad b = \frac{v_H q_H}{{}_m vq}\ {}_H^\pi\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}_L^\pi\ {}_L :
$$

$^{w}_{H}$

$$\frac{@L}{@\;{}^{w}_{H}} =$$

$$= \sum_{k} l_k \left( -\frac{c'_w}{M(\;)\,w_k} \frac{dz^w_k}{d\;{}^w_H} \right) + \sum_{m} q_m \left( -\frac{c'_\pi}{\frac{M\;\theta}{\theta}\;{}_m} \frac{dz^\pi_m}{d\;{}^w_H} \right)$$

$$+ \frac{dz^w_H}{d\;{}^w_H} \frac{1}{M(\;)\,w_H} l_H c'_w \quad \frac{z^w_H}{M(\;)\,w_H} \quad + \;{}_H l_H c''_w \quad \frac{z^w_H}{M(\;)\,w_H} \quad \frac{1}{M(\;)\,w_H} \frac{dz^w_H}{d\;{}^w_H}$$

$$+ \frac{dz^w_L}{d\;{}^w_H} \frac{1}{M(\;)\,w_L} l_L c'_w \quad \frac{z^w_L}{M(\;)\,w_L} \quad + \;{}_L l_L c''_w \quad \frac{z^w_L}{M(\;)\,w_L} \quad \frac{1}{M(\;)\,w_L} \frac{dz^w_L}{d\;{}^w_H}$$

$$+ \frac{dz^\pi_H}{d\;{}^\pi_H} \frac{1}{\frac{M\;\theta}{\theta}\;{}_H} q_H c'_\pi \quad \frac{z^\pi_H}{\frac{M\;\theta}{\theta}\;{}_H} \quad + v_H q_H c''_\pi \quad \frac{z^\pi_H}{\frac{M\;\theta}{\theta}\;{}_H} \quad \frac{1}{\frac{M\;\theta}{\theta}\;{}_H} \frac{dz^\pi_H}{d\;{}^\pi_H}$$

$$+ \frac{dz^\pi_L}{d\;{}^\pi_L} \frac{1}{\frac{M\;\theta}{\theta}\;{}_L} q_L c'_\pi \quad \frac{z^\pi_L}{\frac{M\;\theta}{\theta}\;{}_L} \quad + v_L q_L c''_\pi \quad \frac{z^\pi_L}{\frac{M\;\theta}{\theta}\;{}_L} \quad \frac{1}{\frac{M\;\theta}{\theta}\;{}_L} \frac{dz^\pi_L}{d\;{}^\pi_L}$$

$$+ \left[ \sum_{k} \frac{l_k}{M(\;)\,w_k} \frac{dz^w_k}{d\;{}^w_H} \quad M(\;) + (\sum_{k} l) M(\;) \left( \frac{\sum_m \frac{q_m}{\frac{M(\theta)}{\theta}\pi_m} \frac{dz^\pi_m}{d\tau^w_H}}{\sum_k l} - \frac{(\sum_m vq) \sum_k \left( \frac{l_k}{M\;\theta\;\;w_k} \frac{dz^w_k}{d\tau^w_H} \right)}{(\sum_k l)^2} \right) \right] (a+b)$$

[illegible]

$$+ \;(\sum_{k} l) M(\;)$$

fi

$$_H l_H c''_w \frac{z^w_H}{M(\ ) w_H} \quad \frac{1}{M(\ ) w_H}\frac{dz^w_H}{d\ ^w_H} + \ _L l_L c''_w \frac{z^w_L}{M(\ ) w}$$ [redacted]

$$+v_H q_H c''_\pi \frac{z^\pi_H}{\frac{M\,\theta}{\theta}\ _H} \quad \frac{1}{\frac{M\,\theta}{\theta}\ _H}\frac{dz^\pi_H}{d\ ^w_H} + v_L q_L c''_\pi \frac{z^\pi_L}{\frac{M\,\theta}{\theta}\ _L}$$ [redacted]

$$+ (\ _k\ l)M(\ ) \left[ \frac{k\left(\frac{l_k}{M\,\theta\ w_k}\frac{dz^w_k}{d\tau^w_H}\right.}{_k\ l} + \frac{M'(\ )}{M(\ )} \quad \frac{m}{} \right.$$ [redacted] $$\left. \frac{\frac{l_k}{M\,\theta\ w_k}\frac{dz^w_k}{d\tau^w_H}}{_k\ l} \right] (a+b) =$$

$$= \ _H l_H c''_w \frac{z^w_H}{M(\ ) w_H} \quad \frac{1}{M(\ ) w_H}\frac{dz^w_H}{d\ ^w_H}\frac{z^w_H}{z^w_H}\frac{M(\ )(}{c'_w(z^w_H=}$$ [redacted]

$$+ \ _L l_L c''_w \frac{z^w_L}{M(\ ) w_L} \quad \frac{1}{M(\ ) w_L}\frac{dz^w_L}{d\ ^w_H}\frac{z^w_L}{z^w_L}\frac{M(\ )(1-\ ^w_L)w_L}{c'_w(z^w_L = M(\ ) w_L)}$$

$$+v_H q_H c''_\pi \frac{z^\pi_H}{\frac{M\,\theta}{\theta}\ _H} \quad \frac{1}{\frac{M\,\theta}{\theta}\ _H}\frac{dz^\pi_H}{d\ ^w_H}\frac{z^\pi_H}{z^\pi_H}\frac{M(\ )}{}\frac{(1-\ ^\pi_H)\ _H}{c'_\pi(z^\pi_H = \ _H)}$$

$$+v_L q_L c''_\pi \frac{z^\pi_L}{\frac{M\,\theta}{\theta}\ _L} \quad \frac{1}{\frac{M\,\theta}{\theta}\ _L}\frac{dz^\pi_L}{d\ ^w_H}\frac{z^\pi_L}{z^\pi_L}\frac{M(\ )}{}\frac{(1-\ ^\pi_L)\ _L}{c'_\pi\left(z^\pi_L = \frac{M\,\theta}{\theta}\ _L\right.}$$

$$+ (\ _k\ l)M(\ ) \left[ \quad k\left(\frac{l_k}{M\,\theta\ w\mathrm{wH}}\right.\right.$$

$$= \quad {}_{H} l_{H} \frac{1}{''^{w}_{H}} \frac{dz^{w}_{H}}{d \quad {}^{w}_{H}} \frac{1}{z^{w}_{H}}$$

$_H^\pi$, $_L^\pi$

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad )^{1 - \quad _L^w}$$

F

$$= 1 - \frac{\left[\begin{array}{l} \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) w_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) w_{HL} \quad {}^{w}_{H} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) w_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) w_{LL} \quad {}^{w}_{L} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) {}_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) {}_{HL} \quad {}^{\pi}_{H} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) {}_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) {}_{LL} \quad {}^{\pi}_{L} \end{array}\right]^{-}}{\frac{-\tau^w_H}{\varepsilon^w_H} w_H + \frac{-\tau^w_L}{\varepsilon^w_L} w_L + \frac{-\tau^\pi_H}{\varepsilon^\pi_H} {}_{H} + \frac{-\tau^\pi_L}{\varepsilon^\pi_L} {}_{L}};$$

$$= \left(1 - \frac{E_{(s)} w_{Hs} {}^{w}_{H} + E_{(s)} w_{Ls} {}^{w}_{L} + E_{(s)} {}_{sH} {}^{\pi}_{H} + E_{(s)} {}_{sL} {}^{\pi}_{L}}{\frac{-\tau^w_H}{\varepsilon^w_H} w_H + \frac{-\tau^w_L}{\varepsilon^w_L} w_L + \frac{-\tau^\pi_H}{\varepsilon^\pi_H} {}_{H} + \frac{-\tau^\pi_L}{\varepsilon^\pi_L} {}_{L}}\right)^{-}; \qquad (64)$$

$s$ $\left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \ldots\right.$

$c'$

$$'' = \frac{1}{\,} = \frac{c''}{c'} = \frac{1}{\, - 1}:$$

$A$, :

.

2 $c = A( \;^{\gamma} + \;^{\beta})$ $\geq 3$, $<$ , $c' = A( \;^{\gamma -} + \;^{\beta -})$,

$$c'' = A( \;( \; - 1) \;^{\gamma - 2} + \;( \; - 1) \;^{\beta - 2}) > 0;$$

$$'' = \frac{\;^{\gamma - 2} + \;^{\beta - 2}}{( \; - 1) \;^{\gamma - 2} + \;( \; - 1) \;^{\beta - 2}}:$$

$A$ , , .

$$\frac{@''}{@} = - \;^{\beta} \;^{\gamma -} ( \; - \;)^2 < 0:$$

N $\geq 3$ $=$ $-1$. . F ,

$< 1$ $= 3$ $= 2$, $> 1$ $= 2$ $= 1$.

$\frac{-\tau_H^w}{\varepsilon_H^w} w_H < \frac{-\tau_L^w}{\varepsilon_L^w} w_L$, (66) -

. H $(1 - \;_H^w) w_H < (1 - \;_L^w) w_L$ $\;_H^w > \;_L^w$

(66) $\;_H^w w_H > \;_L^w w_L$. N $(E_{(m)} \;_{Hm} \;_m^{\pi} - E_{(m)} \;_{Lm} \;_m^{\pi})$. -

$\;_{HH} > \;_{LH}$ $\;_{HL} > \;_{LL}$ $(E_{(m)} \;_{Hm} \;_m^{\pi} - E_{(m)} \;_{Lm} \;_m^{\pi} > 0)$.

H (66) .

$(1 - \;_H^w) w_H > (1 - \;_L^w) w_L$, $\;_H > \;_L$, $''^w_H \leq ''^w_L$. □

*Proof of Proposition 11.*

fi (65).

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(60)-(63)

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(60) (61)

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad) \frac{1 - \quad ^{w}}{''_{w}} w + \quad (w \quad ^{w} + \quad ^{\pi} - (1 - \quad) \bar{R}) \quad =$$

$$= (1 - \quad) \left[ (1 - \quad ^{w}) w + \quad (w \quad ^{w\,''w} + \quad ^{\pi\,''w} - (1 - \quad) \bar{R}^{''w}) \right]$$

$$-$$

(68), (69), P 11.

P 12

(1 − ), (1 − )= ↑,

, $^{\pi}$= $^{w}$ ↓.□