Generating the Hopf Fibration Experimentally in Nematic Liquid Crystals

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$\pi_1(\mathsf{R}P^2)$, while a point boundary, i.e., a hole in the surface, carries a Z charge associated with $\pi_2(\mathsf{R}P^2)$. To illustrate, the surface $\Box_{\hat{z}}$ for a toron is shown in Figs. 1(c) and 1(d). It is the set of all points in the material where the director is perpendicular to $\hat{z}$ and forms a surface that connects the two point defects at the "top" and "bottom" of the toron.

The surfaces, however, do not carry enough information to determine the point charges. In order to capture this

There is an important constraint on the colors that may

$Q/U = \tan(2\ )$, where J is the amplitude of the signal. Here n is an exponent depending on the imaging modality; $n = 4$ for the case of fluorescence confocal microscopy [20,21], $n = 6$ for 3PEF-PM with fluorescence detection without a polarizer [18], and $n = 8$ for coherent anti-Stokes Raman scattering polarizing microscopy with linearly polarized detection collinear with the polarization of excitation light. We then assume that we can shift and normalize the calculated I from the data, so that it takes value from 0 to 1 and the nth root of I gives us $\sin\ $. The shift is justified in this case, as we expect that away from the toron the director is actually normal to the top and bottom surfaces, along the surface normal, and hence $\ = 0$ there. The angle then gives us the angle of the polarization projected to the plane, and we can reconstruct the director $\hat{n}$ from and .

To go from this to the colored surface numerically, we reflect the director field so that it lies in the upper half of the sphere; i.e., if $\cos\ < 0$, we take $\hat{n} \to -\hat{n}$. Using PARAVIEW [19], we then view the isocontour with $n\ $ close to zero. Though one might want to take a slice with $n\ $ zero, the nonorientability of the line field makes it difficult to exclude the artificial ‘‘branch cuts’’ where any reconstruction assigns adjacent grid points to different branches of $\hat{n}$, for example, when $\hat{n}$ happens to be adjacent to a data point of $-\hat{n}$. The downside of our approach is that what should be one surface at $n\ = 0$ is actually two nearby surfaces $n\ = \pm$

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