Hopf Solitons in Helical and Conical Backgrounds of Chiral Magnetic Solids

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and D defining the helical wavelength $\lambda = 2\pi J/D$. w_Z is the Zeeman coupling term, where $\boldsymbol{H}$ is the applied field, M_s is the saturation magnetization, and μ_0 is the vacuum permeability. The magnetocrystalline anisotropy term is $w_a = -K_u(\boldsymbol{m} \cdot \hat{k})^2$ for uniaxial anisotropy and $w_a = -K_c(m_x^4 + m_y^4 + m_z^4)$ for cubic anisotropy, where K_u and $\hat{k}$ are the uniaxial anisotropy strength and axis, and K_c is the cubic anisotropy strength. To make our findings applicable to a broad range of materials (see Supplemental Material [30]), we use dimensionless fields and anisotropy strengths $\tilde{\boldsymbol{H}} = \boldsymbol{H}/H_D$ and $\tilde{K}_{u(c)} = K_{u(c)}/\mu_0 M_s H_D$, where $H_D = D^2/\mu_0 M_S J$ is the critical field along the helical axis that fully unwinds the helical state [30].

Adopting the field configuration of liquid-crystal heliknotons in Ref. [17] as the initial condition of $\boldsymbol{m}(\boldsymbol{r})$, we minimize free energy and find that the individual 3D-localized magnetic heliknotons in the bulk helical background at no fields or anisotropies [Figs. 1(a)–1(d)] as metastable states with energy $E_0 = 8.58J\lambda$ when taking the helical background state as the reference. Preimages of constant $\boldsymbol{m}(\boldsymbol{r})$ corresponding to $\mathbb{S}^2$ points are closed loops interlinking once with other individual preimages. This geometric analysis allows the assignment of the Hopf index $Q = 1$

field $\boldsymbol{q}_0$ [Fig. 1(e)]. Preimages of $\mathbb{S}^2$ points of constant polar but different azimuthal angles form deformed tori nested around the preimages of north and south poles [Fig. 1(f)]. The two sets of nested tori are separated by the preimages of points on the equator of $\mathbb{S}^2$, representing the region occupied by the far field. In a heliknoton embedded in a helical background, preimage tori corresponding to points of the same latitude on either hemisphere of $\mathbb{S}^2$ intertransform by a π rotation along $\boldsymbol{q}_0$ with respect to the geometric center of the heliknoton. This symmetry is broken when a magnetic field is applied along $\boldsymbol{q}_0$ and the helical background transitions into the conical state with a cone angle $_{c} = \cos^{-1}\tilde{H}$ [Fig. 1(a)]. As a result of such helical-to-conical transition in the far field, two preimage tori of polar angles $_1$ and $_2$ ($_1 < _c < _2 < \pi/2$) transition from being both coaxial with the north pole's preimage to forming a non-coaxial link of preimage tori, with the overall $\pi_3(\mathbb{S}^2)$ topology of $\boldsymbol{m}(\boldsymbol{r})$ [Fig. 1(g)]. Thus, heliknotons can exist in a conical field background of varying cone angle, though we could stabilize heliknotons only up to $\tilde{H} \approx 0.2$. Beyond this field, the high-energy cost of regions with $\boldsymbol{m}(\boldsymbol{r})$ antiparallel to $\boldsymbol{H}$ overcomes the topological barrier, transforming the $Q = 1$ heliknoton to the topologically trivial conical state through nucleation and propagation of singular defects (Bloch points) [35].

While heliknotons are fully nonsingular structures in $\boldsymbol{m}(\boldsymbol{r})$, nontrivial topology characterizes not only this material field. Singular vortex lines in nonpolar $\boldsymbol{q}(\boldsymbol{r})$ form three mutually linked loops, different from the trefoil-knot vortices of liquid crystal heliknotons [17,30] [Fig. 1(h)]. We also calculate the emergent field $(B_{em})_i \equiv \hbar\ ^{ijk}$

energy than the topologically trivial structures but persist as metastable states within a broad parameter range (colored green in Fig. 3). Within metastability regions, these solitons are often geometrically deformed by fields and anisotropies (Fig. 4) [30]. Interestingly, this stretching preserves topology parameters, can be large ($\Delta E_{23}/k_B T \approx 10$ and $T = 200$ K for FeGe) or comparable ($\Delta E_{23}/k_B T \approx 0.7$ and $T = 25$ K for MnSi) to thermal energy (see Supplemental Material [30]). With the formation of tetramer and octamer configurations, the free energy per heliknoton is further reduced. Within the heliknoton oligomer, the isosurfaces of perturbation in $\boldsymbol{q}(\boldsymbol{r})$ of individual heliknotons join into a single surface and the overall Hopf index becomes the sum of that of the solitonic constituents [Figs. 5(d) and 5(e)]. Thus, a heliknoton oligomer resembles a single high-charge heliknoton molecule or, in a different analogy, a high-baryon-number nucleus [42]. The complex configuration of the stabilized octamer cannot be straightforwardly expected on the basis of dimer or tetramer configurations, suggesting that the emergent crystalline assemblies of heliknotons could be complex. A systematic study of all possible symmetries and lattice parameters, for different external fields and magnetocrystalline anisotropies, could potentially reveal the energy-minimizing asse. t the emer-

[20] X. Zhang, Y. Zhou, and M. Ezawa, Sci. Rep.