Interaction and co-assembly of optical and topological solitons

Guilhem Poy[1,2], Andrew J. Hess[3], Andrew J. Seracuse[3], Michael Paul[3], Slobodan Žumer[1,4] and Ivan I. Smalyukh[3,5,6]

Solitons are ubiquitous in nature and technology[1]. They are found as water waves, pulses of light, wavefunctions of Cooper pairs in superconducting Josephson junctions, propagating pulses in biomembranes and nervous systems[1–4], models of elementary particles[5] and even cosmological objects like black holes[6–8]. An optical soliton generally refers to an optical waveform that maintains its shape when evolving over long distances and/or times, and even after collisions. This concept now encompasses a broad class of wavepackets with multiple spatial and temporal dimensions[9]. Having a particle-like nature, optical solitons can mutually attract, repel or yield fusion, fission and annihilation in media with nonlocal optical nonlinearities[10].

Many recent studies have focused on optical solitons in liquid crystals (LCs). These soft birefringent media exhibit giant nonlinear and nonlocal optical responses, with a facile reorientation of their optical axis fields under external stimuli, enabling the generation of optical solitons at powers as low as ~1 mW. An archetypal example of optical solitons in LCs is the so-called nematicon, which propagates without diffracting by creating its own waveguide in the optical axis field of the LC[11–13]. Trajectories of nematicons may be modified using optical reflections from dielectric interfaces and various deformed regions of the background optical axis field[14–17]. Their potential for photonics applications has recently been demonstrated with mode transformations[18], bistability[19,20] and soliton-assisted random lasing[21]. Other types of optical solitons in LCs include discrete solitons[22], optothermal and dark solitons in dye-doped LCs[23,24], self-focused beams with fast-evolving polarization states and spin–orbit interactions[25–27], and optical solitons in non-frustrated[28] or frustrated chiral LCs[29]. At the same time, chiral LCs are also known to host a fascinating variety of topological solitons, like skyrmions, hopfions, torons and fingers[30–34], which correspond to localized and topologically protected patterns of the optical axis embedded within their uniform backgrounds $\mathbf{n}_0$. These robust structures can be created on demand with strong external stimuli[35,36], they are stable without external fields, and they cannot be continuously deformed into the uniform background $\mathbf{n}_0$.

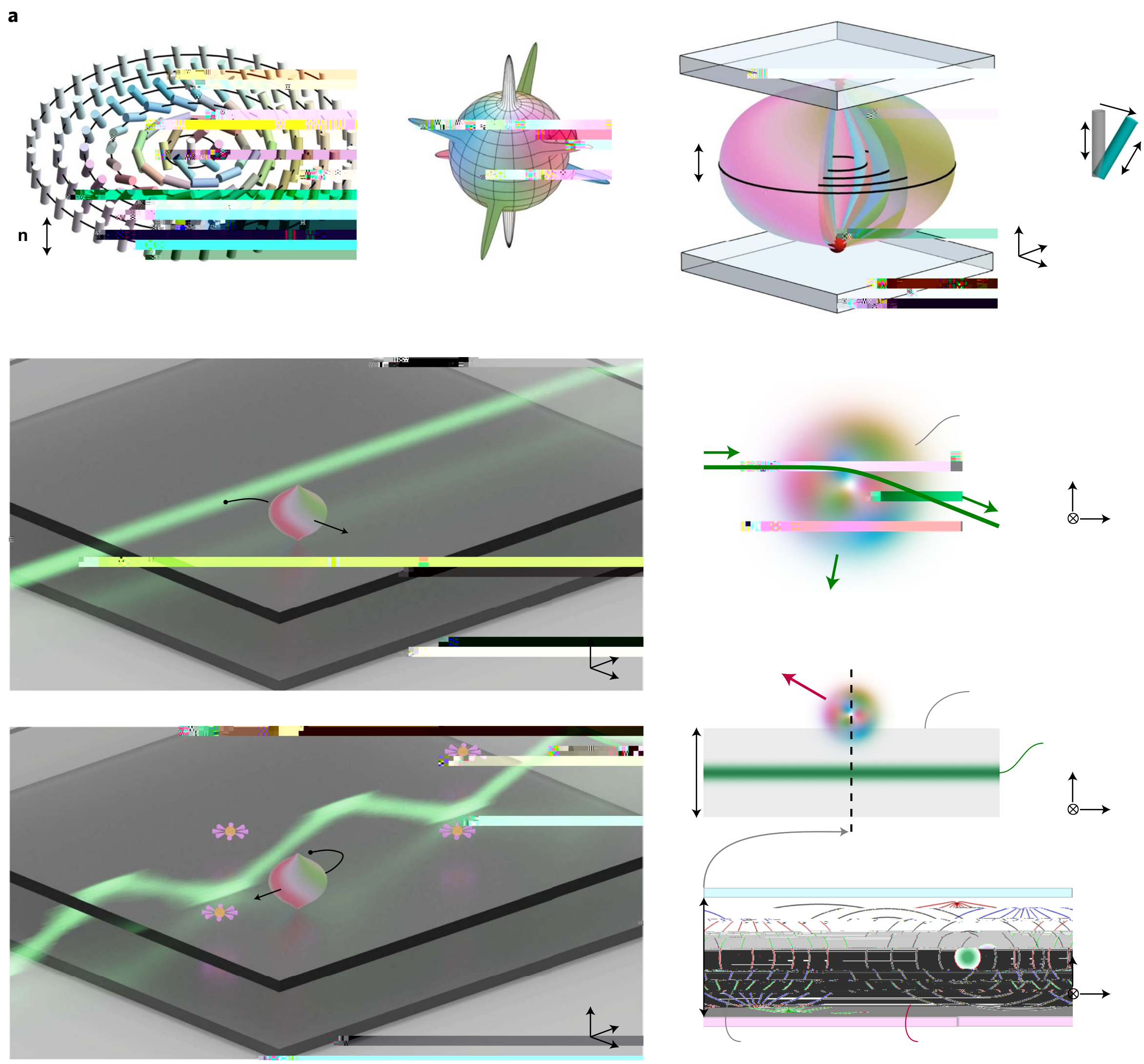
Discoveries of different types of laser light and matter interactions have had a strong impact on the development of fundamental science and technologies throughout recent history, from laser surgery to laser trapping of tiny particles, to laser cooling of atomic gases, and to the generation of Bose–Einstein condensates. However, to the best of our knowledge, none of these diverse forms of light–matter interactions have exploited the regime when both light and matter take solitonic embodiments.

Here, we experimentally discover and theoretically explain the fascinating interactions between topological solitons and two classes of optical solitons in LCs, thus showing how the particle-like nature of optical solitons enables optomechanical interactions with topological solitons. By focusing on a regime where each type of soliton is not perturbed too much, we experimentally characterize these interactions and theoretically explain them in an elegant and accurate manner with an effective Langevin equation that accounts for optical forces similar to the ones of optical traps, as well as nonlocal effects associated with the light-induced realignment of the optical axis field. The surprising findings within this new regime of light–matter interactions reveal that the interplay of nonlinear effects that stabilizes these different solitons can lead to the self-assembly of topological solitons beside the optical solitons. Such interactions yield exceptionally rich types of behaviour that may find practical uses ranging from nonlinear optics to nanophotonic devices and spatial light–matter co-patterning.

Results

Physics of inter-solitonic interactions and co-assembly. Our experimental investigations show that topological solitons in the optical axis of a uniaxial chiral nematic LC can not only be attracted or pushed away from the optical solitonic beam's axis but can also co-assemble in highly nontrivial ways (Fig. 1c) when forming

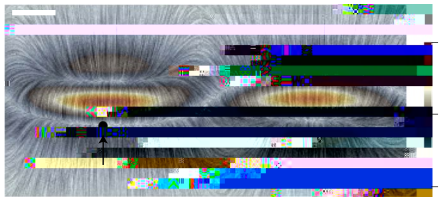
[1]Faculty of Mathematics and Physics, University of Ljubljana, Ljubljana, Slovenia. [2]Laboratoire Charles Coulomb, Univ Montpellier, CNRS, Montpellier, France. [3]Department of Physics, University of Colorado, Boulder, CO, USA. [4]Jožef Stefan Institute, Ljubljana, Slovenia. [5]Chirality Research Center, Hiroshima University, Higashi Hiroshima, Japan. [6]Renewable and Sustainable Energy Institute, National Renewable Energy Laboratory and University of Colorado, Boulder, CO, USA. e-mail: ivan.smalyukh@colorado.edu

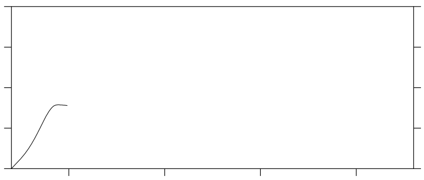


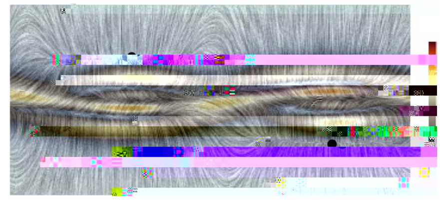
one-dimensional arrays of topological solitons localizing on the sides of the optical soliton. We experimentally reveal that this behaviour stems from the elaborate dynamical trajectories of topological solitons navigating their way when guided by particular optical solitons (Fig. 1c, bottom). What is the physical underpinning of this unexpected behaviour?

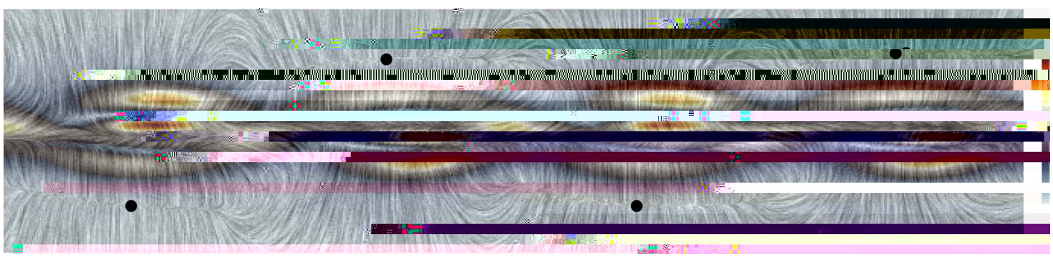
Our unwound chiral LC system is confined between glass plates, imposing an optical axis orientation $\mathbf{n}_0$ normal to the surfaces (see Methods). These samples host localized patterns of optical axis embedding an emblematic example of a topological soliton—the baby skyrmion—whose optical axis field is shown in Fig. 1a and whose name refers to Skyrme's topological solitons used to describe subatomic particles with different baryon numbers[5]. It covers twice the order parameter space of nonpolar unit vectors $\mathbf{n}$ with antipodal symmetry $\mathbf{n} \leftrightarrow -\mathbf{n}$, that is, antipodal points on a sphere (Fig. 1a, right). To emphasize the topological and nonpolar properties of these optical axis patterns, we introduce a colour scheme that associates an optical axis orientation with a colour. Our topological colouring is used for all cylindrical-glyph-based or continuous-colour plots of topological solitons and is detailed in Supplementary Section 2C, where we explain all the subtleties. For the skyrmion of Fig. 1a, one can easily check that white corresponds to the far-field optical axis $\boldsymbol{n}_0$ and that the primary colours blue, red and green (associated with tilted cylinders) appear twice. Each of these colours is associated with the antipodal peaks on the sphere of Fig. 1a. The two-dimensional structure of Fig. 1a corresponds to the mid-sample plane of the three-dimensional structure of Fig. 1b, which shows a few isosurfaces with fixed angles between $\mathbf{n}$ and $\mathbf{n}_0$. The quantity $\boldsymbol{\delta}\mathbf{n}^{\mathrm{TS}}$ corresponds to the deviation of the optical axis field $\mathbf{n}^{\mathrm{TS}}$ with respect to the far-field optical axis $\mathbf{n}_0 \equiv \mathbf{e}_x$ (unit vector of x-axis on Fig. 1) imposed by the confining plates and fully defines this class of topological structures called torons[32], where 'TS' refers to topological solitons. Although more complex torons[32] and other solitons[30–34] can be realized, our study in this work focuses solely on the simplest type of elementary toron depicted in Fig. 1. Further

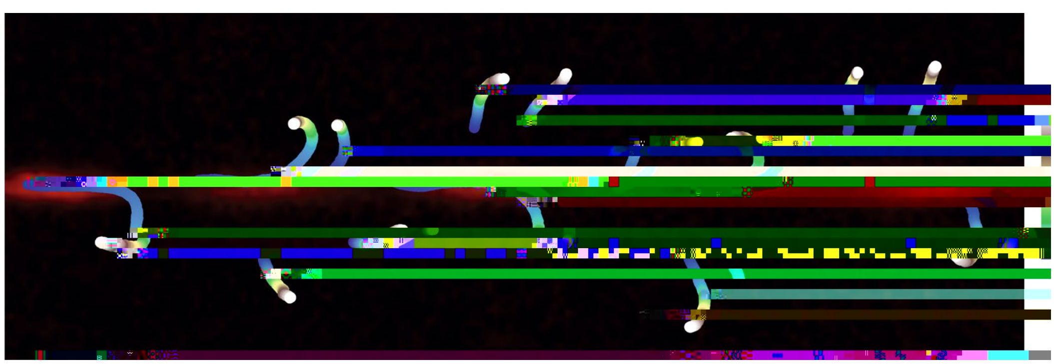
The force $\mathbf{F}^{L}$ is directly proportional to the opposite of the averaged deflection of light's momentum $\mathbf{\Delta p}$ (see Fig. 1d) and is there-











(see Supplementary Section 3) as well as their rescaled intensities $\tilde{\mathbf{F}}^{\alpha} \equiv \mathbf{F}^{\alpha}\text{-P}$ (with P the beam power and α

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Methods

Preparation of LC samples with topological solitons. Each experimental cell consists of an LC layer sandwiched between two glass plates treated for perpendicular boundary conditions (homeotropic anchoring)[51,52]. To de ne

(Fig. 1e). We numerically calculate this interaction energy using the simulated optical axis patterns $\mathbf{n}^{TS}$ and $\mathbf{n}^{EM}$ for different shifts $\mathbf{R}$ of the toron with respect the optical soliton, and then deduce the nonlocal optical force field as $\mathbf{F}^{NL} = -\nabla_{\mathbf{R}} G$. Simulations of the internal structure of topological and optical solitons were carried out using the numerical methods described elsewhere[29,60] and detailed in Supplementary Section 2.

Data availability

All data and postprocessing scripts are available from the Zenodo repository (https://doi.org/10.5281/zenodo.6394431). Polarized optical microscopy simulations were performed using the open-source software Nemaktis (https://github.com/warthan07/Nemaktis and https://doi.org/10.5281/zenodo.4695959).

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